Phase Transition of 4D Simplicial Quantum Gravity with $U(1)$ Gauge Field *

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The phase transition of 4D simplicial quantum gravity coupled to $U(1)$ gauge fields is studied using Monte-Carlo simulations. The phase transition of the dynamical triangulation model with a vector field ($N_V = 1$) is smooth compared with pure gravity($N_V = 0$). The node susceptibility (χ) is studied by the finite size scaling method. At the critical point, the node distribution has a sharp peak in contrast to the double peak in pure gravity. From the numerical results, we expect that 4D simplicial quantum gravity with $U(1)$ vector fields has a higher order phase transition than 1st order, which means the possibility to take the continuum limit at the critical point.

1. Introduction

The phase structure of 4D pure simplicial quantum gravity has been intensively investigated. In 4D pure gravity, two distinct phases are known. For small values of the bare gravitational coupling constant the system is in the so-called elongated phase, which has the characteristics of a branched polymer. For large values of the bare gravitational coupling constant it is in the so-called crumpled phase. Numerically, the phase transition between the two phases has been shown to be 1st order [1,2]. As a result, it is difficult to construct a continuum theory. Our next step is to investigate an extended model of 4D quantum gravity. Recently the phase structure of the extended models of 4D quantum gravity has been studied numerically [3,4]. In a model with vector fields, the intermediate phase has been observed between the crumpled phase and the elongated phase. We consider the possibility of a continuum limit at the critical point. In order to investigate the phase transition, we measure the finite size scaling of node susceptibility (χ),

$$\chi = (< N_0^2 > - < N_0 >^2)/N_4, \tag{1}$$

as well as the scaling property of the mother boundary in the case of 4D simplicial quantum gravity coupled to one gauge field ($N_V = 1$).

2. Phase Diagram with Gauge Fields

We consider the partition function of simplicial gravity coupled to $U(1)$ gauge fields. The total action is $S = S_{EH} + S_{pl}$. We use the Einstein-Hilbert action for gravity,

$$S_{EH}[\Lambda, G] = \int d^4x\sqrt{g}(\Lambda - \frac{1}{G}R), \tag{2}$$

where Λ is the cosmological constant and G is Newton's constant. We use discretized action for gravity,

$$S_{EH}[\kappa_2, \kappa_4] = \kappa_4 N_4 - \kappa_2 N_2, \tag{3}$$

where $\kappa_2 \sim 1/G$, κ_4 is related to Λ and N_i is the number of i-simplices. We use the plaquette action for $U(1)$ gauge fields [3],

$$S_{pl} = \sum_{t_{ijk}} o(t_{ijk})[A(l_{ij}) + A(l_{jk}) + A(l_{ki})]^2, \tag{4}$$

where l_{ij} denotes a link between vertices i and j, t_{ijk} denotes a triangle with vertices i, j and

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k, $A(l_{ij})$ denotes the $U(1)$ gauge field on a link l_{ij}, and $o(t_{ijk})$ denotes the number of 4-simplices sharing triangle t_{ijk}. We consider that a partition function of gravity with N_V copies of the $U(1)$ gauge fields is

$$Z(\kappa_2, \kappa_4, N_V) = \sum_{N_4} e^{-\kappa_4 N_4} \sum_{t(2D)\in T(4D)} e^{\kappa_2 N_2} \prod_{N_V} \int \prod_{l \in t(2D)} dA(l) e^{-S_{pl}}. \quad (5)$$

We sum over all 4D simplicial triangulation, $T(4D)$, in order to carry out a path integral over the metric. Here, we fix the topology with S^4. Numerically, in the case of adding vector fields, three phases have been found [3,4]. A schematic phase diagram has been shown in ref.[4]. An intermediate region is called the smooth phase between these two transition points,[1] κ_2^c and κ_2^o. We expect that the phase transition at κ_2^c is continuous and leads to continuum limit of 4D quantum gravity. Now, let us notice the transition at κ_2^c in the case of $N_V = 1$.

3. Numerical Analysis of Phase Transition ($N_V = 1$)

In this section we report on two numerical observations: the node susceptibility (χ) and the histogram of N_0. In Fig.1 we plot the node susceptibility (χ) as a function of κ_2 with volume $N_4 = 16K, 24K$ and $32K$, respectively. The node susceptibility (χ) has a peak value at the critical point (κ_2^c). We measure the peak value in each size. As a finite size scaling, the peak value (χ_{max}) and the width of peak ($\delta\kappa_2$) grow as N_4 in power. The susceptibility exponents, Δ and Γ, are defined by [2]

$$\chi_{max} \propto N_4^{\Delta} \quad (\delta\kappa_2 \propto N_4^{-\Gamma}). \quad (6)$$

From the numerical result (Fig.1), we obtain the susceptibility exponent $\Delta = 0.4(1)$, ($\Gamma \sim 0.5(3)$). These values are apparently smaller than 1. In Fig.2 we show the histogram of N_0 at the critical point ($\kappa_2^c = 1.37147(1)$) with the size of

[1]The usual phase transition point (κ_2^c) and the obscure phase transition point (κ_2^o) are defined in ref.[4]

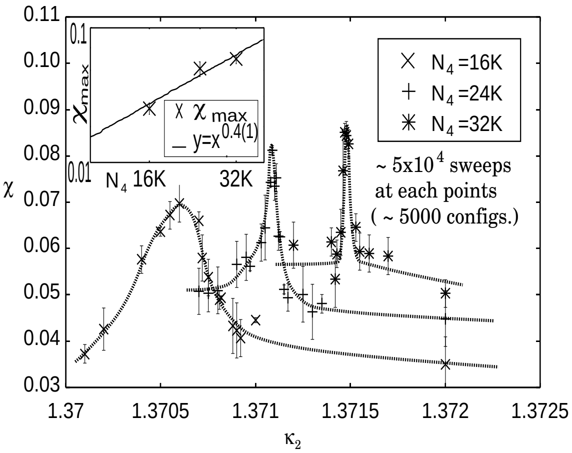

Figure 1. Node susceptibility (χ) plotted versus the coupling constant (κ_2) and χ_{max} plotted versus N_4 with log-log scales for $N_V = 1$.

$N_4 = 32K$. In pure gravity, a double peak structure has been found[1,2]. The fact shows that the phase transition is 1st order. However, in the case of $N_V = 1$, the double peak structure disappears. We then consider that the phase transition between the crumpled phase and the smooth phase may be continuous, not 1st order.

4. Scaling Property of 4D Simplicial Quantum Gravity

In this section we discuss the scaling structure of 4D DT manifolds, focusing on the scaling structure of boundaries in 4D Euclidean space-time using the concept of geodesic distances. In order to discuss the universality of the scaling relations, we assume that the boundary volume distribution ($\rho(x, D)$) is a function of a scaling variable, $x = V/D^{\alpha}$, with the scaling parameter α in the analogy of 2D quantum gravity. Here, V denotes the volume of the boundary and D is the geodesic distance. The expectation value of the boundary three-dimensional volume appearing at distance D has been introduced in ref.[5],

$$< V^{(3)} >= \frac{1}{N} \int_{v_0}^{\infty} dV \, V \rho(x = V/D^{\alpha}, D), \quad (7)$$

where v_0 denotes UV cut-off of the boundary volume and N is the normalization factor. If the boundary volume has a scaling property with the

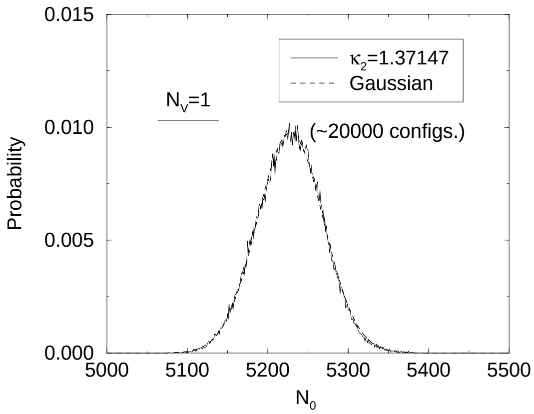

Figure 2. Histogram of node(N_0) at the critical point ($\kappa_2 = 1.37147$) with the size of $N_4 = 32K$.

universal distribution $\rho(x, D)$ and $v_0 \to 0$,

$$< V^{(3)} >\sim D^{\alpha}. \tag{8}$$

Then, we obtain a finite fractal dimension, $d_f = \alpha + 1$, with the fractal dimension d_f. We measure the volume of the mother boundary as a function of D. The mother boundary is defined by the boundary having the largest tip volume. In Fig.3 we plot the mother boundary volume, $< V^{(3)} >$, at the critical point with a size of $N_4 = 32K$. As a result, the mother boundary volume shows scaling and we obtain the scaling parameter $\alpha = 3.7(5)$. Then, we can estimate the fractal dimension ($d_f = 4.7(5)$). On the other hand, we measure the Hausdorff dimension, $d_H = 4.6(2)$. Both results are consistent. Thus, we expect that the boundary volume distribution has a scaling property in the sense of the manifold at different distances from a given 4-simplex and looks exactly the same after a proper rescaling of the boundary volume.

5. Summary and Discussions

Let us summarize the main points made in the previous sections. For the phase transition, we show the finite size scaling at the critical point and the histogram of node. From the numerical results, the phase transition is smooth in contrast to pure gravity. Also we show the scaling property of the mother boundary, where the scaling parameter is consistent with the fractal dimension. We expect that the boundaries have a fractal structure and the universality of the scaling relations. From a modification of the Balls-in-Boxes model[6], we expect that the simplicial quantum gravity coupled to matter fields will have the possibility of a continuous phase transition in $N_V \geq 1$. We expect that the existence of genuine 4D quantum gravity at the critical point, κ_2^c, with the vector fields.

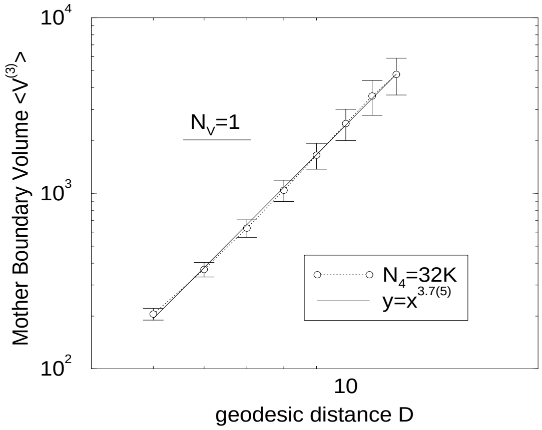

Figure 3. Power fit to mother boundary three-dimensional volume $< V^{(3)} >$ at the critical point ($\kappa_2 = 1.37147$) with the size of $N_4 = 32K$ with log-log scales.

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